Dynamically Screened Elastic Collisions in Nonideal Plasmas

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The dynamic screening effects on elastic electron-ion collisions are investigated in nonideal plasmas. The second-order eikonal method with the impact parameter analysis is employed to obtain the eikonal phase as a function of the impact parameter, collision energy, thermal energy, and Debye length. The result shows that the eikonal phase decreases with increasing the thermal energy. It is also found that the dynamic screening effects on the eikonal phase are more significant for large impact parameters. The total eikonal cross section is also found to be decreased with increasing the thermal energy. It is important to note that the eikonal cross section and the eikonal phase including the dynamic screening effects are found to be greater than those including the static screening effects.

Key words: Dynamic Screening; Nonideal Plasmas.

It is known that the elastic electron-ion collision is one of the major atomic collision processes due to its applications in many areas of physics. Recently, the plasma diagnostics using the atomic collision and radiation processes [1 – 6] in plasmas have paved new ways to investigate various plasma parameters. It is known that the description of a charged particle system is one of the most interesting problems in modern physics. The plasma described by the ordinary Debye-Hückel potential is known as the ideal plasma since the average interaction energy between charged particles is found to be smaller than the average kinetic energy of a particle [7]. However, multiparticle correlation effects caused by simultaneous interaction of many charged particles have to be taken into account with an increase of the plasma density. In these circumstances, the interaction potential may not be represented by the ordinary Debye model due to the strong collective effects of nonideal particle interactions [8 – 10]. Then, the elastic electron-ion collisions in nonideal plasmas would be different from those described by the ordinary Yukawa-type Debye interaction potential. The quantum mechanical eikonal method [11, 12] has been widely used in many collision processes. Especially, this eikonal analysis has a great advantage since the generalized continuum wave function and the collision cross section can be obtained in terms of the eikonal phase with the effective interaction potential. Thus, in this paper we investigate the dynamic screening effects on elastic electron-ion collisions in nonideal plasmas using the second-order eikonal method. The modified Debye form of the effective interaction potential [10] taking into account the dynamic screening effect is applied to describe electron-ion interactions in nonideal plasmas. The impact parameter analysis is applied to obtain the eikonal phase as a function of the scaled impact parameter, Debye length, thermal energy, and collision energy.

From the Lippmann-Schwinger equation [13]

$$\Psi_k(\mathbf{r})\Phi_k(\mathbf{r}) + \frac{2\mu}{\hbar^2}\int \mathrm{d}^3\mathbf{r}' G_0(\mathbf{r},\mathbf{r}')V(\mathbf{r}')\Psi_k(\mathbf{r}'), \quad (1)$$

where $\Psi_k(\mathbf{r})$, $\Phi_k(\mathbf{r})$, and $G_0(\mathbf{r},\mathbf{r}')$ are the solution of the Schrödinger equation, the solution of the homogeneous equation, and the Green's function, respectively, the following functions are derived:

$$(\nabla^2 + k^2)\Psi_k(\mathbf{r}) = \frac{2\mu}{\hbar^2}V(\mathbf{r})\Psi_k(\mathbf{r}), \quad (2)$$

$$(\nabla^2 + k^2)\Phi_k(\mathbf{r}) = 0, \quad (3)$$

$$(\nabla^2 + k^2)G_0(\mathbf{r},\mathbf{r}') = \delta^3(\mathbf{r},\mathbf{r}'). \quad (4)$$

Here k $(= \sqrt{2\mu E}/\hbar)$ is the wave number, E the collision energy, μ the reduced mass of the collision system, $V(\mathbf{r})$ the interaction potential, and $\delta^3(\mathbf{r},\mathbf{r}')$ the Dirac delta function. Using the free outgoing Green's function [11], $G_0^{(+)}(\mathbf{r},\mathbf{r}')$ $(= -\mathrm{e}^{-\mathrm{i}k|\mathbf{r}-\mathbf{r}'|}/4\pi|\mathbf{r}-\mathbf{r}'|),$

the solution of (2) can be expressed in the form:

$$\Psi_k^{(+)}(\mathbf{r}) = \frac{1}{(2\pi)^{3/2}} \mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{r}} \cdot \left[1 + \frac{2\mu}{\hbar^2} \int \mathrm{d}^3\mathbf{r}' \mathrm{e}^{-ik\cdot(\mathbf{r}-\mathbf{r}')} G_0^{(+)}(\mathbf{r}-\mathbf{r}') V(\mathbf{r}')\right]. \quad (5)$$

The validity condition of the eikonal method is known as $|V|/E < 1$ [13], where $|V|$ is a typical strength of the interaction potential. Using the cylindrical coordinate system such that $\mathbf{r} = \mathbf{b} + z\hat{n}$, where $\mathbf{b}$ is the impact parameter, $\hat{n}$ is the unit vector perpendicular to the momentum transfer Δ $(\equiv \mathbf{k}_\mathrm{i} - \mathbf{k}_\mathrm{f})$, $\mathbf{k}_\mathrm{i}$ and $\mathbf{k}_\mathrm{f}$ are the incident and final wave vectors, respectively, the eikonal scattering amplitude f_E is found to be

$$f_\mathrm{E} = -\frac{\mu}{2\pi\hbar^2} \int \mathrm{d}^3\mathbf{r}\, \mathrm{e}^{\mathrm{i}\Delta\cdot\mathbf{r}} V(\mathbf{r}) \cdot \exp\left[-\frac{\mathrm{i}\mu}{\hbar^2 k_\mathrm{i}} \int_{-\infty}^{z} \mathrm{d}z' V(\mathbf{b}, z')\right]. \quad (6)$$

The total eikonal collision cross section is then given by

$$\sigma_\mathrm{E} = \int \mathrm{d}\Omega |f_\mathrm{E}|^2 = 2\pi \int \mathrm{d}b\, b |\exp[\mathrm{i}\chi_\mathrm{E}(k,b)] - 1|^2, \quad (7)$$

where $\chi(k,b)$ $[= \chi_1(b)/k + \chi_2(b)/k^3]$ is the total eikonal phase, $|\mathbf{k}_\mathrm{i}| = |\mathbf{k}_\mathrm{f}| = k$, $\chi_1(b)$ and $\chi_2(b)$ are, respectively, the first- and second-order eikonal phases [12]:

$$\chi_1(b) = -\frac{\mu}{\hbar^2} \int_{-\infty}^{\infty} \mathrm{d}z V(r), \quad (8)$$

$$\chi_2(b) = -\left(\frac{\mu}{2\hbar^2}\right)^2 \int_{-\infty}^{\infty} \mathrm{d}z V(r) \left[V(r) + r \frac{\mathrm{d}}{\mathrm{d}r} V(r)\right]. \quad (9)$$

Very recently, the analytic form [10] of the modified Debye potential including the dynamic screening effect was obtained in nonideal plasmas. Using this effective potential model, the dynamic interaction potential V_ei between the projectile electron and the target ion with charge Ze in nonideal plasmas can be obtained by

$$V_\mathrm{ei}(r,v) = -\frac{Ze^2}{r} \exp[-r/r_0(v)], \quad (10)$$

where $r_0(v)$ $[= r_\mathrm{D}(1 + v^2/v_\mathrm{th}^2)^{1/2}]$ is the modified screening length, r_D the Debye length, v the collision velocity, v_th $(= \sqrt{k_\mathrm{B}T/m})$ the thermal velocity, k_B the Boltzmann constant, T the plasma temperature, and m the electron mass. In (10), the velocity dependence of the plasma screening length can be understood since the projectile velocity is smaller than the electron thermal velocity, and the dynamic plasma screening effect becomes the static plasma screening effect, i. e., $r_0(v) \to r_\mathrm{D}$. After some manipulations using the effective interaction potential (10) and the impact parameter analysis with the definition of the modified Bessel function of the second kind of order n [14]

$$K_n(x) = \frac{\pi^{1/2}}{(n-1/2)!} \left(\frac{x}{2}\right)^n \int_1^{\infty} \mathrm{d}t\, \mathrm{e}^{-xt} (t^2-1)^{n-1/2}, \quad (11)$$

the first- and second-order eikonal phases are found to be, respectively,

$$\frac{1}{k}\chi_1(\bar{b}) = \frac{2}{\bar{E}^{1/2}} K_0\left[a_\mathrm{D}(1 + \bar{E}/\bar{E}_\mathrm{th})^{-1/2} \bar{b}\right], \quad (12)$$

$$\frac{1}{k^3}\chi_2(\bar{b}) = \frac{a_\mathrm{D}}{2\bar{E}^{3/2}} (1 + \bar{E}/\bar{E}_\mathrm{th})^{-1/2} \cdot K_0\left[2a_\mathrm{D}(1 + \bar{E}/\bar{E}_\mathrm{th})^{-1/2} \bar{b}\right], \quad (13)$$

where $\bar{b}$ $(\equiv b/a_Z)$ is the scaled impact parameter, a_Z $(= a_0/Z)$ the Bohr radius of the hydrogenic ion with nuclear charge Ze, a_0 $(= \hbar^2/me^2)$ the Bohr radius of the hydrogen atom, a_D $(\equiv a_Z/r_\mathrm{D}$ the scaled reciprocal Debye length, $\bar{E}$ $(\equiv mv^2/2Z^2Ry)$ the scaled collision energy, Ry $(= me^4/2\hbar^2 \approx 13.6$ eV) the Rydberg constant, and $\bar{E}_\mathrm{th}(\equiv k_\mathrm{B}T/2Z^2Ry)$ the scaled thermal energy. Here, the factor $a_\mathrm{D}(1 + \bar{E}/\bar{E}_\mathrm{th})^{1/2}$ indicates the dynamic screening effects on the eikonal phases. The approach given in this work may be related to the semi-classical investigation of spectral line shapes given by Griem [15]. The total eikonal cross section σ_E for the elastic electron-ion collision in units of πa_Z^2 is then found to be

$$\sigma_\mathrm{E}(a_\mathrm{D}, \bar{E}, \bar{E}_\mathrm{th})/\pi a_Z^2 = 2\int_0^{r_\mathrm{D}/a_Z} \mathrm{d}\bar{b}\,\bar{b} \left| \exp\left[\frac{2\mathrm{i}}{\bar{E}^{1/2}} K_0[a_\mathrm{D}(1+\bar{E}/\bar{E}_\mathrm{th})^{-1/2}\bar{b}] + \frac{\mathrm{i}a_\mathrm{D}}{2\bar{E}^{3/2}} (1+\bar{E}/\bar{E}_\mathrm{th})^{-1/2} K_0[2a_\mathrm{D}(1+\bar{E}/\bar{E}_\mathrm{th})^{-1/2}\bar{b}]\right] - 1\right|^2, \quad (14)$$

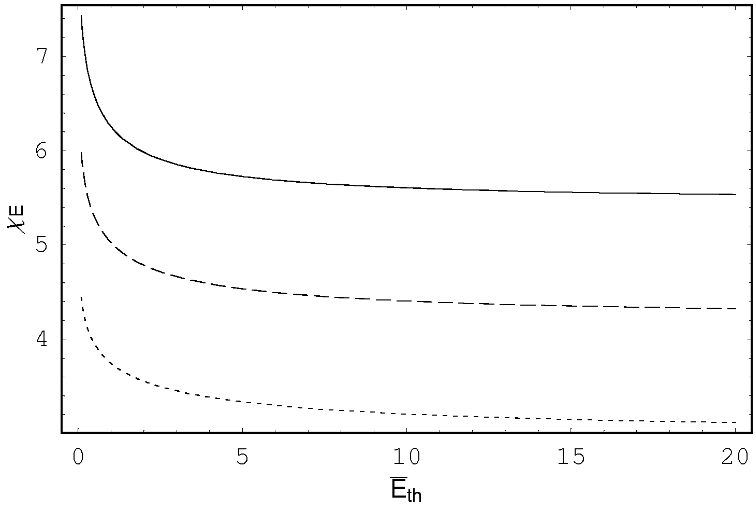

Fig. 1. The eikonal phase (χ_E) as a function of the scaled thermal energy $(\bar{E}_{th})$ at $\bar{b} = 1$ for $a_D = 0.01$. The solid line is the eikonal phase including the dynamic screening effects for $\bar{E} = 3$. The dashed line is the eikonal phase including the dynamic screening effects for $\bar{E} = 5$. The dotted line is the eikonal phase including the dynamic screening effects for $\bar{E} = 10$.

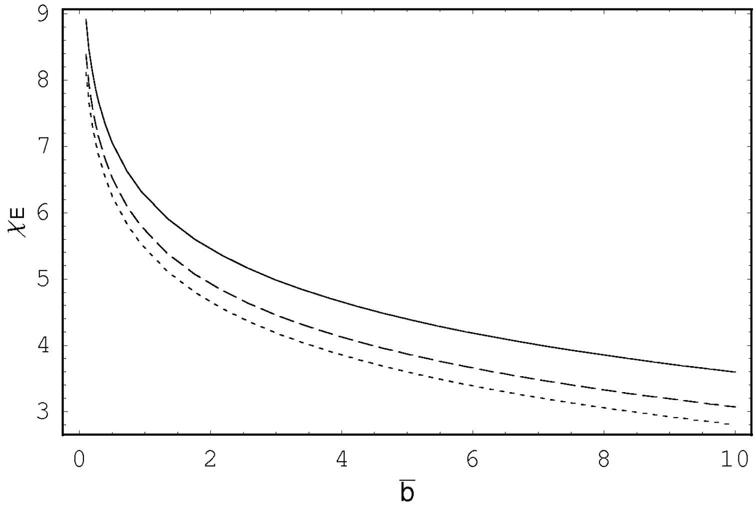

Fig. 2. The eikonal phase (χ_E) as a function of the scaled impact parameter $(\bar{b})$ for $a_D = 0.01$. The solid line is the eikonal phase including the dynamic screening effects for $\bar{E} = 3$ and $\bar{E}_{th} = 1$. The dashed line is the eikonal phase including the dynamic screening effects for $\bar{E} = 3$ and $\bar{E}_{th} = 5$. The dotted line is the eikonal phase including the static screening effects for $\bar{E} = 3$.

where the upper bound (r_D/a_Z) in the integral represents the cutoff screening length. If we neglect the dynamic screening effects, i. e., using the ordinary Debye potential, the total eikonal electron-ion collision cross section σ'_E becomes

$$\sigma'_E(a_D,\bar{E})/\pi a_Z^2 = 2\int_0^{r_D/a_Z} d\bar{b}\bar{b}\left|\exp\left[\frac{2i}{\bar{E}^{1/2}}K_0(a_D\bar{b}+\frac{ia_D}{2\bar{E}^{3/2}}K_0(2a_D\bar{b})\right]-1\right|^2. \tag{15}$$

In order to explicitly investigate the dynamic screening effects on the total eikonal electron-ion collision cross section in nonideal plasmas, we consider $\bar{E} > 1$ since the eikonal method is known to be valid for high-energy projectiles [11]. More consistent discussions of the effect of dynamical screening on collision cross section and dynamical collision frequency based on a quantum statistical approach were given by using the Green function approach [16 – 18]. Figure 1 represents the total eikonal phase as a function of the scaled thermal energy. As we see in this figure, the eikonal phase is found to decrease with increasing thermal energy. Figure 2 provides the comparison between the dynamic screening effects and the static screening effects on the eikonal phase. From this figure, it is found that the eikonal phase including the dynamic screening effects is greater than that including the static screening effects. Figure 3 shows the three-

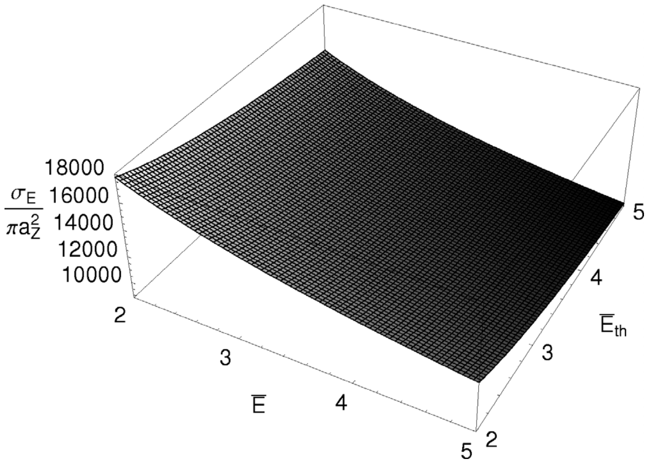

Fig. 3. The three-dimensional plot of the scaled eikonal cross section $(\sigma_{\rm D}/\pi a_Z^2)$ as a function of the scaled collision energy $(\bar{E})$ and the scaled thermal energy $(\bar{E}_{\rm th})$ for $a_{\rm D} = 0.01$.

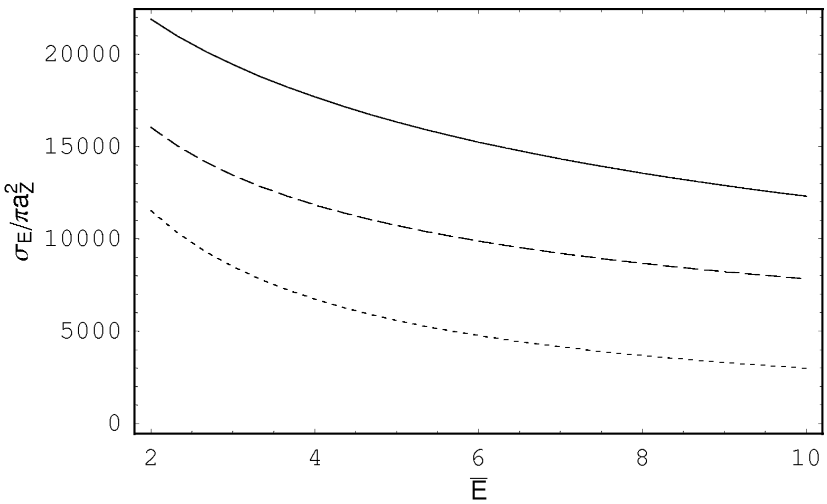

Fig. 4. The eikonal cross section $(\sigma_{\rm E})$ in units of πa_Z^2 as a function of the scaled collision energy $(\bar{E})$ for $a_{\rm D} = 0.01$. The solid line is the eikonal cross section including the dynamic screening effects for $\bar{E}_{\rm th} = 1$. The dashed line is the eikonal cross section including the dynamic screening effects for $\bar{E}_{\rm th} = 3$. The dotted line is the eikonal cross section including the static screening effects.

dimensional plot of the total eikonal cross section for the elastic electron-ion collision in units of πa_Z^2 as a function of the collision energy and the thermal energy. Figure 4 shows the comparison between the dynamic screening effects and the static screening effects on the total eikonal cross section. From these figures, it is also found that the total eikonal cross section decreases with increasing thermal energy. In addition, the eikonal cross section including the dynamic screening effects is found to be greater than that including the static screening effects. It can be understood that when the velocity of the plasma electron is comparable to or smaller than the velocity of the projectile electron, the static plasma screening formula is not reliable since the projectile electron polarizes the surrounding plasma electrons. In these circumstances the dynamic motion of the plasma electron has to be considered to investigate the plasma screening effects on the collision processes. Thus, the eikonal cross section including the dynamic screening effects is greater than that including the static screening effects due to the weakening of the plasma screening effect in high projectile energies. These results provide useful information concerning the dynamic screening effects on the elastic collision in nonideal plasmas.

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